The authors would like to express appreciation to S. A. Khristianovich, A. T. Onufriev, and M. D. Shcherbin for their interest in the study and for many useful discussions.

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MECHANISM OF REDISTRIBUTION OF AN ALKALI ADDITION IN THE CHANNEL OF AN MHD GENERATOR

D. I. Lamden and M. B. Reznikov

UDC 538.4

It is of great interest to study diffusion and mass-transfer processes that take place in an easily-ionized alkali addition (usually potassium) introduced into the channel of an MHD generator to increase the conductivity of the plasma, since these processes to a large extent determine the operating characteristics of the generator. Flow in the channel is appreciably nonisothermal in character, which leads to variability of the composition of the addition over the cross section. At temperatures of about 3000°K, typical of the flow core, the addition is mainly in the form of potassium ions, and the quantity of KOH molecules and potassium ions is one order of magnitude lower. With approach toward the relatively cold walls ($T \sim 1000$-2000°K), the concentration of atomic and ionized potassium begins to decrease due to an increase in the concentration of KOH molecules. Thus, at temperatures of about 2000-1500°K, the addition is mainly in the form of KOH. Finally, a decrease in temperature to below 1500°K is accompanied by the beginning of the reaction of KOH with the dioxide in the combustion products to form the carbonate K_2CO_3. The diffusion countercurrents which develop here lead to a nonuniform distribution of potassium as an element across the channel due to a difference in the diffusion coefficients of the components. The addition is also redistributed as a result of thermal diffusion and absorption of the addition on the walls. It was shown in [1, 2] that the drift of potassium ions in an electrical and magnetic field may lead to significant redistribution of the addition over the cross section of an MHD channel and, in particular, to an increase in its concentration near the cathode and a decrease in same as the anode. The goal of the present work is to obtain general equations describing the redistribution of an addition under the influence of the group of mechanisms discussed above and to analyze the contribution of each mechanism.

1. Flow in an MHD channel is turbulent both as a result of the natural turbulence of the combustion products and due to the development of turbulent boundary layers at the walls. Since all of the effects described below are important in these boundary layers, we can examine just the transverse diffusion flow of the addition as an element, which is equal to

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Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 5, pp. 76-85, September-October, 1985. Original article submitted June 19, 1984.

$$j = -ND\frac{\partial c}{\partial y} - cNF + j_i, \tag{1.1}$$

where

$$c = \sum_k c_k g_k, \quad D = \sum_k D_k g_k \xi_k, \quad \xi_k = c_k/c,$$

$$F = \sum_k D_k g_k (\partial \xi_k/\partial y) + \sum_k D_k^{\mathrm{m}} \alpha_k^T \xi_k g_k (\partial \ln T/\partial y).$$

Here, j and c are the flow and the relative concentration of the addition as an element (potassium, for the sake of definiteness); N is the concentration of molecules of the buffer gas of combustion products; c_k and g_k are the relative concentrations of the neutral components of the addition and the quantity of potassium atoms in their molecules; $D_k = D_k^m + D^t$ is the total diffusion coefficient (both molecular and turbulent); α_k^T is the thermal diffusion constant; T is temperature; j_i is the diffusion current of ions. The latter should be sought from the solution of the system

$$\begin{gathered}
j_i = -ND_i\frac{\partial c_i}{\partial y} + \mu_i N c_i (E + v_x B), \\
j_e = -ND_e\frac{\partial c_e}{\partial y} - \mu_e N c_e (E + v_x B), \\
J = e(j_i - j_e), \quad \frac{\partial E}{\partial y} = \frac{e}{\varepsilon_0} N (c_i - c_e),
\end{gathered} \tag{1.2}$$

where j_e is the flow of electrons; J is the current density; D_i, D_e and μ_i, μ_e are the coefficients of diffusion and mobility of the ions and electrons; E and B are the electrical and magnetic fields; v_x is the longitudinal component of the flow velocity; ε_0 is the dielectric constant. Transforming (1.2), we divide the first two terms of the equation by μ_i and μ_e, respectively, add them, and insert j_e and $(c_i - c_e)$ from the last two equations into the resulting expression. As a result, ignoring the small quantity μ_i/μ_e relative to unity, we have

$$j_i = \frac{J}{e}\frac{\mu_i}{\mu_e} + \frac{\varepsilon_0}{e}(E + v_x B)\frac{\partial E}{\partial y}\mu_i - ND_i\frac{\partial c_i}{\partial y} - N\frac{D_e}{\mu_e}\mu_i\frac{\partial c_e}{\partial y}. \tag{1.3}$$

Here, the first term on the right side corresponds to the ion current density in a quasineutral plasma, while the second describes the deviation of the ion current from the quasineutral value due to the action of strong electrical fields in the electrode regions of the space charge (i.e., in those regions where quasineutrality is disturbed). As was shown in [2], the last two terms in (1.3), connected with concentration diffusion, can be ignored because they make a small contribution on the order of max $(c_i/c, c_e/c)$ in the relations obtained below for the distribution of the addition.

The sought distribution is described by the following equations in the boundary-layer approximation:

$$\begin{gathered}
Nv_x\frac{\partial c}{\partial x} + Nv_y\frac{\partial c}{\partial y} = -\frac{\partial}{\partial y}\left(-ND\frac{\partial c}{\partial y} - cNF + j_i\right), \\
c(x, \infty) = c_\infty = \text{const}, \quad \left.\left(-ND\frac{\partial c}{\partial y} - cNF + j_i\right)\right|_{y=0} = j_w,
\end{gathered} \tag{1.4}$$

where j_w is the flow of the addition to the wall; c_∞ is the concentration of the element in the core of the flow. Due to the linearity of this equation (j_i can be considered independent of c in the first approximation), we will formally represent its solution in the form $c = c^0 + \Delta c^{\mathrm{CRA}}$, where c^0 satisfies the equation

$$\begin{gathered}
Nv_x\frac{\partial c^0}{\partial x} + Nv_y\frac{\partial c^0}{\partial y} = -\frac{\partial}{\partial y}\left(-ND\frac{\partial c^0}{\partial y} - c^0 NF\right), \\
c^0(x, \infty) = c_\infty = \text{const}, \\
(-ND\partial c^0/\partial y - c^0 NF)|_{y=0} = j_w,
\end{gathered} \tag{1.5}$$

i.e., it is the solution of an ordinary mass-transfer problem in the absence of the current. The CRA (current redistribution of the addition) being described Δc^{CRA} obviously satisfies Eq. (1.4) with the boundary conditions

$$\Delta c^{\mathrm{CRA}}(x, \infty) = 0, \quad (-ND\partial\Delta c^{\mathrm{CRA}}/\partial y - \Delta c^{\mathrm{CRA}} NF + j_i)|_{y=0} = 0,$$

i.e., it is independent of the flow of addition to the wall. We write Δc^{CRA} in the form

$$\Delta c^{\mathrm{CRA}} = -\Phi \int_{y}^{\infty} \frac{j_i\,dy}{ND\Phi} + \beta,\ \Phi = \exp\left[-\int_{0}^{y} (F/D)\,dy\right], \tag{1.6}$$

where the first term is the solution of (1.4) for Δc^{CRA} when we ignore the convective terms [the left side of (1.4)] and β is a correction for convection. After insertion of (1.3) into (1.6), we have

$$\begin{gathered} \Delta c^{\mathrm{CRA}} = \Delta c_{\mathrm{n}}^{\mathrm{CRA}} + \Delta c_{\mathrm{c}}^{\mathrm{CRA}} + \beta, \\ \Delta c_{\mathrm{n}}^{\mathrm{CRA}} = -\Phi \int_{y}^{\infty} \frac{J}{e} \frac{\mu_i}{\mu_e} \frac{dy}{ND\Phi}, \quad \Delta c_{\mathrm{c}}^{\mathrm{CRA}} = -\Phi \int_{y}^{\infty} \frac{\varepsilon_0 (E + v_x B)}{eND\Phi} \frac{\partial E}{\partial y} \mu_i dy. \end{gathered} \tag{1.7}$$

Here, $\Delta c_{\mathrm{n}}^{\mathrm{CRA}}$ and $\Delta c_{\mathrm{c}}^{\mathrm{CRA}}$ are the contributions of the quasineutral region and the space-charge layer to the CRA obtained when convection is ignored. The quantity β is determined by the following equation after the insertion of (1.6) into (1.4):

$$\begin{gathered} \left(Nv_x \frac{\partial}{\partial x} + Nv_y \frac{\partial}{\partial y}\right)\left(\Delta c_{\mathrm{n}}^{\mathrm{CRA}} + \Delta c_{\mathrm{c}}^{\mathrm{CRA}} + \beta\right) = -\frac{\partial}{\partial y}\left(-ND\frac{\partial \beta}{\partial y} - \beta NF\right), \\ \beta(x, \infty) = 0, \quad \left(-ND\frac{\partial \beta}{\partial y} - \beta NF\right)\Big|_{y=0} = 0. \end{gathered} \tag{1.8}$$

Since the space-charge region lies within the viscous sublayer, where convection is insubstantial, in calculating β we can omit the term $\Delta c_{\mathrm{c}}^{\mathrm{CRA}}$ in (1.8). To evaluate β, we will examine a hypothetical laminar plane-parallel flow with a velocity distribution conforming to a "one-seventh" law, i.e. [3],

$$v_x(\theta) = v_\infty \theta^{1/7}, \quad v_y = 0, \quad \theta \equiv y/\delta(x), \quad \delta(x) = 0.37x\,\mathrm{Re}_x^{-0.2},$$

where $\delta(x)$ is the thickness of the boundary layer; $\mathrm{Re}_x = v_\infty x/\nu^m$ is the Reynolds number (ν^m is the molecular kinematic viscosity). Assuming for simplicity that $\Phi(y) = 1$, $\mathrm{Sc} = \mathrm{Sc}^t = 1$ (molecular and turbulent Schmidt numbers), and $(J/e)(\mu_i/\mu_e) = \mathrm{const}$, we find the following if we assume that the frictional stress is constant across the boundary layer and is equal to its value on the wall:

$$\begin{gathered} D = 0.545 \frac{v_\infty \delta^2}{x} \theta^{6/7}, \\ \Delta c_{\mathrm{n}}^{\mathrm{CRA}} = -12.8 \frac{J}{e} \frac{\mu_i}{\mu_e} \frac{1}{Nv_\infty}\left(1 - \theta^{1/7}\right)\left(\frac{x}{\delta}\right) \equiv \Psi(\theta)\left(\frac{x}{\delta}\right). \end{gathered}$$

Inserting this expression into (1.8) and assuming that $\beta = (x/\delta)\chi(\theta)$, we write the following ordinary differential equation for

$$\chi'' + \chi'((6/7)\theta^{-1} + 1{,}47\theta^{2/7}) - 0{,}367\theta^{-5/7}\chi + (1{,}47\Psi'\theta^{2/7} - 0.367\Psi\theta^{-5/7}) = 0.\ \chi(1) = 0.\ \chi'(0) = 0,$$

i.e., the problem is a similarity problem for the given flow. Figure 1 shows graphs of $\Psi(\theta)/\Psi(0)$ $(=\Delta c_{\mathrm{n}}^{\mathrm{CRA}}(x, y)/\Delta c_{\mathrm{n}}^{\mathrm{CRA}}(x, 0))$ and $\chi(\theta)/\Psi(0)$ $(=\beta(x, y)/\Delta c_{\mathrm{n}}^{\mathrm{CRA}}(x, 0))$, from which it follows that the contribution of β to Δc^{CRA} is small in the most important boundary regions (about 10% of $\Delta c_{\mathrm{n}}^{\mathrm{CRA}}$). On the other hand, it follows from the estimates presented below that CRA is unimportant in the regions where $\Delta c_{\mathrm{n}}^{\mathrm{CRA}} \sim \beta$. Thus, the value of β in (1.6) can be ignored when calculating Δc^{CRA}.

As concerns the "mass-transfer" term c^0 from (1.5), for convenience in the subsequent analysis we will represent it in the form

$$c^0 = c_\infty \Phi(y)/\Phi(\infty) + \Delta c^j,$$

where the first term describes the distribution of c^0 when the convective terms are ignored and when $j_w = 0$. The quantity Δc^j satisfies the same boundary conditions as does Δc^{CRA} (with the replacement of j_i by $-j_w$). Since the constant $-j_w$ can be inserted under the differentiation sign in the right side of (1.5), Δc^{CRA} and Δc^j formally satisfy the same equations. Thus in this case we can also assume with good accuracy that

$$\Delta c^j = \Phi \int_{y}^{\infty} \frac{j_w}{ND\Phi}\,dy. \tag{1.9}$$

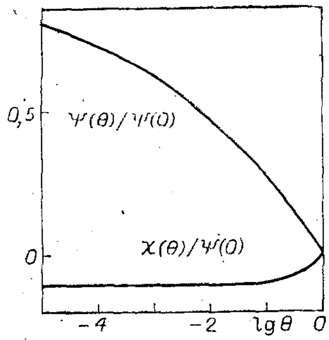

Fig. 1

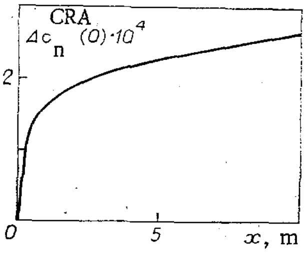

Fig. 2

Finally, we have the following for the complete redistribution of the addition due to all of the effects examined above

$$c(y) = c_\infty \Phi(y)/\Phi(\infty) + \Delta c^j + \Delta c_n^{CRA} + \Delta c_c^{CRA} \tag{1.10}$$

The value of j_w needed to calculate Δc_j should be determined on the basis of specific boundary conditions. In writing these conditions [with the use of (1.10)], we can obtain an equation relative to j_w. Insertion of the resulting j_w into (1.9) also gives the solution to the problem.

2. Let us take a closer look at the function $\Phi(y)$ from (1.6), which describes the effect of thermal diffusion and the difference in the diffusion coefficients of the components on the distribution. For convenience, we write it, using (1.1), in the form of the product of the diffusion and thermal diffusion terms:

$$\Phi = \Phi_D \Phi_T = \exp\left[-\int_0^y \frac{\sum_k D_k g_k (d\xi_k/dy)}{\sum_k D_k g_k \xi_k}\, dy\right] \exp\left[-\int_0^y \frac{\sum_k D_k^m g_k \xi_k \alpha_k^T (d\ln T/dy)}{\sum_k D_k g_k \xi_k}\, dy\right]. \tag{2.1}$$

In the simplest case, when the diffusion coefficients of different neutral components are equal (and their thermal diffusion coefficients are also equal), Φ, by virtue of the equality $\sum_k g_k \xi_k = 1$, turns out to be a function only of y and T = T(y), i.e., (1.10) turns out to be the sought solution in quadratures. The same result is obviously obtained when the coefficients of diffusion and thermal diffusion are different, but there is local chemical equilibrium, and it is assumed that no molecules containing two or more potassium atoms (such as K_2CO_3) are formed [given these assumptions, $\xi_k = \xi_k(T(y))$]. The last condition can in principle be dropped if we assume that the boundary layer has a two-layer structure (purely turbulent and purely viscous regions), ignore thermal diffusion, and assume that the molecular diffusion coefficients of the neutral components have the same temperature dependence (see [1] for more details on this).

At a certain distance from the wall, the characteristic time of diffusion $\tau_D \sim y^2/D$ becomes shorter than the chemical relaxation time. Thus the assumption regarding chemical equilibrium is invalid here. This complicates the problem because now the determination of ξ_k requires numerical solution of a system of differential equations describing diffusion and nonequilibrium chemical reactions. However, if $\Phi(y)$ differs little from unity, then information on the regime of the chemical reactions is unnecessary to calculate the distribution of elemental potassium (1.10). The condition $\Phi(y) \approx 1$ is satisfied with good accuracy due to the relatively small difference between the diffusion coefficients of the components and the freezing of chemical reactions near the wall in an MHD channel.

We will show this by using a two-layer model of the distribution of the transport coefficients:

$$D_k = \begin{cases} D^t, & y > \delta_{\text{л}}, \\ D_k^m(0)\,\varphi(T/T_w), & y < \delta_r, \end{cases} \tag{2.2}$$

where δ_r is the thickness of the viscous sublayer; T_w is the wall temperature; D^t and $\varphi(T/T_w)$ are general functions for all of the components. It follows from (2.1) that in the viscous sublayer

TABLE 1

$k-m$	K—N_2	K—H_2O	KOH—N_2	KOH—H_2O
$D_{km}^{m}(T_0), \frac{m^2}{sec}$	$3{,}05\cdot10^{-4}$	$3{,}93\cdot10^{-4}$	$4{,}95\cdot10^{-4}$	$3{,}46\cdot10^{-4}$
γ	1,68	1,82	1,85	2,16

$$\Phi_D(y) = \sum_k D_k^m(0)\, g_k \xi_k(0) \Big/ \sum_k D_k^m(0)\, g_k\, \xi_k(y),\ y < \delta_r. \tag{2.3}$$

Table 1 shows values of the constants D_{km}^m and γ in the formula

$$D_{km}^m = D_{km}^m(T_0)\,(T/T_0)^\gamma\,(p_0/p_m), \tag{2.4}$$

approximating the results of calculations of binary coefficients of diffusion of the addition in a first approximation of the Champion–Enskog theory [4] (the parameters of molecular interaction were determined by Brokaw's method [5]). In (2.4), $p_0 = 10^5$ Pa, $T_0 = 2000°K$, and p_m is the partial pressure of the m-th component of the buffer gas. With compositions of the combustion products typical of MHD channels, the coefficient of diffusion of the potassium atoms turns out to be lower than the coefficient for the KOH molecules. In accordance with (2.3), this leads to an increase in Φ_D with increasing distance from the wall (since in this case ξ_K increases and ξ_{KOH} decreases). The maximum possible value of $\Phi_D(\delta_r) = D_{KOH}^m(0)/D_K^m(0) \approx 1.4$ is reached when $\xi_K(0) = 0$, $\xi_{KOH}(\delta_r) = 0$ [obviously, $\Phi_D(0) = 1$]. However, for actual conditions, we have the typical relations $\xi_K(\delta_r) \sim \xi_{KOH}(\delta_r)$, $\xi_K(0) \approx 0.2$, $\xi_{KOH}(0) \approx 0.8$ (at $T_w = 2000°K$), which leads to the estimate $\Phi_D(\delta_r) = 1.1$, i.e., the effect turns out to be fairly weak.

The diffusion coefficients of the components are equal in the turbulent region, and it follows from (2.1) and the condition $\sum_k g_k\xi_k = 1$ that

$$\Phi_D(y) = \Phi_D(\delta_r),\ y > \delta_r. \tag{2.5}$$

Returning to Eq. (2.3), we note that the disturbance of local chemical equilibrium near the wall reduces the change in ξ_K and ξ_{KOH} across the viscous sublayer (the reactions are "quenched"), so that the deviation of Φ_D from unity turns out to be even smaller.

Now proceeding to the evaluation of Φ_T, we note that, in accordance with (2.1), the change in Φ_T in the turbulent region proves to be insubstantial because $D^t \gg D^m$. Thermal diffusion leads to accumulation of the addition in the cold wall regions, these regions containing the heaviest and largest molecules in the combustion products [4]. Table 2 shows values of the thermal diffusion constants α^T calculated in a first approximation of the Champion–Enskog theory [4] for the repulsion potential $\varkappa/r^6$ (the constants $\varkappa$ were determined from the data in Table 1 on the diffusion coefficients at T = 2000°K).

It is not possible to obtain such a simple approximate relation as (2.3) for Φ_T, but in this case there is no need to. In fact, it follows from the result for a two-component mixture

$$\Phi_T(y) = [T(y)/T_w]^{-\alpha^T}$$

that with characteristic values of the ratios $T(\delta_r)/T_w \sim 1.1$–1.2, thermal diffusion leads to an increase in the concentration of the addition near the wall by a total of 1-5%, i.e., this effect can be ignored in many cases.

It should be noted that since $\Phi_T < 1$ and $\Phi_D > 1$, the above effects are partially compensated. Thus we can take $\Phi \approx 1$. As indicated above, satisfaction of this condition allows us to calculate Δc^{CRA} (as well as c^0) without regard to the chemical composition of the addition.

3. Now proceeding to the CRA, we note that this effect was examined in detail in [1] for a quasineutral plasma [Δc_n^{CRA} in (1.7) in the special case of a stabilized turbulent flow. Thus, here we make a generalization to the case of flow with boundary layers, having used the two-layer scheme (2.2) for the purpose of simplification.

Using Eqs. (2.3) and (2.5) for Φ_D in the viscous and turbulent regions (thermal diffusion is ignored), we obtain the following from (1.7), assuming $(J/e)(\mu_i/\mu_e)$ = const:

TABLE 2

$k-m$	K—N$_2$	K—H$_2$O	KOH—N$_2$	KOH—H$_2$O
α^T	0,085	0,19	0,071	0,27

$$\Delta c_n^{CRA}(y) = \int_y^\infty \frac{dy}{ND^t}\frac{J}{e}\frac{\mu_i}{\mu_e}, \quad y > \delta_r; \tag{3.1}$$

$$\Delta c_n^{CRA}(y) = \int_{\delta_r}^\infty \frac{dy}{ND^t}\frac{J}{e}\frac{\mu_i}{\mu_e} + \frac{1}{\sum_k D_k^m(0)\, g_k \xi_k(y)} \int_y^{\delta_r} \frac{dy}{N\varphi(y)}\frac{J}{e}\frac{\mu_i}{\mu_e}, \quad y < \delta_r. \tag{3.2}$$

It is obvious that (3.1) and the first integral in (3.2) are determined by the hydrodynamic characteristics of the flow, while the second term in (3.2) also depends on the chemical composition — which varies over the cross section. It follows from (3.2) that the complete redistribution of the addition across the boundary layer

$$\Delta c_n^{CRA}(0) = \int_{\delta_r}^\infty \frac{dy}{ND^t}\frac{J}{e}\frac{\mu_i}{\mu_e} + \int_0^{\delta_r} \frac{dy\,(J/e)\,(\mu_i/\mu_e)}{N\sum_k D_k^m(y)\, g_k \xi_k(0)} = \int_0^\infty \frac{dy}{ND_0}\frac{J}{e}\frac{\mu_i}{\mu_e} \tag{3.3}$$

is determined by the composition of the addition near the wall (D_0 is the coefficient of diffusion of the element for a constant boundary composition), which is usually known beforehand [usually, $\xi_{KOH}(0) \approx 1$]. Thus, calculation of Δc_n^{CRA} does not require information on chemical reactions and can be used to determine the Sherewood number Sh, characterizing the rate of mass transfer of a one-component nonreacting addition in the buffer gas:

$$j_w = -N(\infty)D^m(\infty)\mathrm{Sh}(c(\infty) - c(0))/x, \tag{3.4}$$

where, in accordance with [6, 7], the physical quantities are referred to conditions in the core of nonisothermal turbulent flow (in the flow cooling regime). Comparing (3.4) and (1.9) (with $\Phi = 1$), we have

$$\int_0^\infty \frac{dy}{ND} = \frac{x}{N(\infty)\, D^m(\infty)\, \mathrm{Sh}}; \tag{3.5}$$

$$\Delta c_n^{CRA}(0) = \frac{J}{e}\frac{\mu_i}{\mu_e}\frac{x}{N(\infty)\, D^m(\infty)\, \mathrm{Sh}}, \tag{3.6}$$

where the current density J can be considered a free parameter, since it was shown in [8] that the CRA in the quasineutral region has a slight effect on the volt—ampere characteristics (VAC) of the channel. The similarity between heat and mass transfer allows us to calculate Sh with well-known heat-transfer formulas such as $\mathrm{Sh} = 0.0296\mathrm{Re}_x^{0.8}\mathrm{Sc}^{0.43}$ [9], where Re_x and Sc are the Reynolds and Schmidt numbers (determined in regard to conditions in the flow core). Figure 2 shows the dependence of $\Delta c_n^{CRA}(0)$ on the longitudinal coordinate x for the following characteristic values of the parameters: $T_\infty = 3000°K$, $v_\infty = 500$ m/sec, $\nu_\infty = D_\infty^m = 8.4 \cdot 10^4$ m^2/sec, $J = 10^4$ A/m^2, $\mu_e/\mu_i = 150$, $p = 10^5$ Pa. The effect obviously has different signs on the anode and cathode: The concentration is reduced at the anode and increased at the cathode. As can be seen from Fig. 2, the difference may reach 10% with $c(\infty) = 5 \cdot 10^{-3}$.

In calculating the contribution of the space-charge region to the CRA [the term Δc_c^{CRA} in (1.7)], we can ignore the small term $v_x B$ compared to the field E, which is substantially larger in the present case. Another simplification is connected with the fact that this region lies inside the viscous sublayer. Using the Einstein relation $\mu_i = eD_i^m/kT$ (D_i^m is the coefficient of molecular diffusion of the ions and k is the Boltzmann constant) and taking the same temperature dependence for D_i^m as for D_k^m [see (2.2)], we insert (2.3) into the expression for Δc_c^{CRA} (1.7) and, after abbreviating, we obtain

$$\Delta c_c^{CRA}(y) = \frac{\varepsilon_0}{2p} E^2(y) \frac{D_i^m(0)}{\sum_k D_k^m(0)\, g_k \xi_k(y)} = \frac{\varepsilon_0}{2p} E^2(y) \frac{D_i^m(y)}{D^m(y)}, \tag{3.7}$$

where p is the pressure of the buffer gas [the weak field $E(\infty)$ is ignored]. It follows from (3.7) that calculating of Δc_c^{CRA} requires knowledge of the distribution of E — and, thus, the VAC of the electrode layer. Calculation of the latter is a complicated problem by itself,

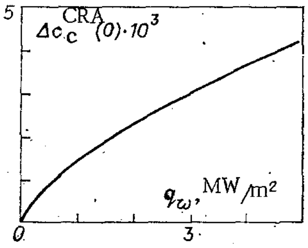

Fig. 3

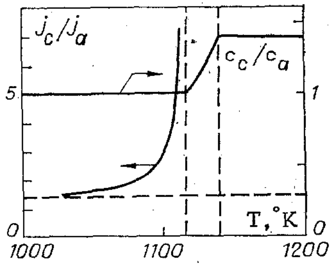

Fig. 4

requiring consideration of heat release, emission, and different ionization mechanisms (see [10, 11], for example). It is obvious that CRA should affect the VAC, so that a self-adjoint problem of calculating Δc_c^{CRA} and E arises. It was shown in [10], however, that even a substantial variation in the distribution of potassium atoms does not have a significant effect on the VAC, i.e., the problem can be considered uncoupled, and well-known results for E obtained without allowance for CRA can be used to calculate (3.7). If there is an empirically or theoretically established VAC for the cathode layer (the value of Δc_c^{CRA} in the anode layer is small due to the weakness of the electrical fields), then for the determination of E(0) — required to calculate $\Delta c_c^{CRA}(0)$ (i.e., the redistribution across the entire layer) — it is possible to use the familiar formula (see [12], for example)

$$E(0) = (3JV_c/\varepsilon_0\mu_i)^{1/3}, \tag{3.8}$$

where V_c is the potential drop in the charged region. Although Eq. (3.8) was obtained for the simplest electrode-layer model, it is evidently universal in character and remains valid for other, more complicated models. For characteristic values $J = 10^4$ A/m², $V_c = 100$ V, $\mu_i(0) = 1.8\cdot 10^{-3}$ m²/(V·sec), $p = 10^5$ Pa, $\mu_i(0)kT_w/eD^m(0) = 1$, we obtain the estimate $\Delta c_c^{CRA}(0) = 1.5\cdot 10^{-3}$. With $c_\infty = 6\cdot 10^{-3}$, the increase in the concentration of the addition at the cathode as a result of the above-examined effect is 25%, i.e., the effect is fairly strong.

It should be noted that the product JV_c in (3.8) is the Joule heating capacity (per unit area of the electrode). At the same time, it was shown in [10] that electrothermal instability of the diffusion discharge and its transition to a contracted discharge occurs when $JV_c \sim q_w$, where q_w is the convective flow of heat to the wall undisturbed by heat liberation. Thus, use of this estimate permits determination of the dependence of Δc_c^{CRA} on q_w for the most favorable regime of MHD generator operation — at the limit of stability of the diffusion discharge. It follows from the form of this relation shown in Fig. 3 that, for different values of q_w, the difference in the concentrations between the cathode and core due to Δc_c^{CRA} may reach 70%.

Due to the proposed division of the problem of calculating the CRA and VAC, the quasineutral and nonquasineutral terms in (1.7) are independent and are additive (the division of the plasma into a quasineutral region and a space-charge region is hypothetical in character and was not used in deriving (1.8) and will not be used subsequently). Thus, the total difference in the concentrations between the cathode and anode may reach 80%. This conclusion agrees particularly with the results of certain empirical studies [13, 14] which noted greater breakdown of the cathode than the anode under the influence of the addition. According to the estimates in [14], 30% more of the addition may be absorbed in porous cathodes than in anodes.

4. Absorption of the addition on the wall (diffusion into pores, condensation on the surface) leads to a decrease in its concentration near the electrodes, which is formally described by the term Δc^j in (1.10).

In the case of hot walls ($T_w \sim 2000$°K), ceramic electrodes are usually used. Here, the addition penetrates the pores in these electrodes to a certain depth, condenses, and crystallizes. As a rule, the diffusion currents which are created here are weak and do not have a significant effect on the distribution of the addition in the flow.

A different situation is seen in the case of cold electrodes ($T_w \sim 1100$°K), when the addition (KOH vapor) condenses on the walls of the channel and the condensate can flow downstream. Here, the process is complicated by the fact that the reaction $2KOH + CO_2 \rightarrow K_2CO_3 + H_2O$ takes place in the electrode regions at $T \lesssim 1500$°K, and the potassium carbonate formed, being supersaturated, condenses in the volume and crystallizes (at T = 1173°K) [15]. However, these processes begin so close to the wall ($y \sim 10^{-4}$ m) that the time of diffusion

$\tau_D \sim 10^{-4}-10^{-5}$ sec is to all indications much shorter than the characteristic time of the chemical reaction. Thus, it has no effect on the distribution of KOH molecules and can be ignored when calculating condensation on the wall. This conclusion is supported by the results in [15], where it was found during observation of deposits on walls that the addition is deposited mainly in the form of KOH. We will also assume that the electrodes are impermeable and that the KOH concentration near the electrodes is equal to the saturation concentration $c^{s}_{KOH}(T_w)$ for a given wall temperature T_w [when there is no condensation, in accordance with (1.10), $c(0) = c_\infty/\Phi(\infty) + \Delta c^{CRA}$]. Assuming Δc^{CRA} to be independent of j_w (more accurately, independent of $\Delta c j$), we obtain the following from (1.10)

$$j_w = -\left[c_\infty/\Phi(\infty) + \Delta c^{CRA} - c^{s}_{KOH}(T_w)\right] \Big/ \int\limits_0^\infty \frac{dy}{ND\Phi}$$

$$\text{at} \quad c_\infty/\Phi(\infty) + \Delta c^{CRA} \geqslant c^{s}_{KOH}(T_w), \tag{4.1}$$

$$j_w = 0 \quad \text{at} \quad c_\infty/\Phi(\infty) + \Delta c^{CRA} < c^{s}_{KOH}(T_w).$$

A negative value of j_w in (4.1) is connected with the chosen direction of the y axis – from the wall to the flow. It is evident from (4.1) that the ratio of the flows on the anode and cathode depends on the wall temperature [due to the difference in $\Delta c^{CRA}(0)$ on them – on the cathode, where $\Delta c^{CRA} > 0$, condensation may already have begun, while it remains equal to zero at the cathode, where $\Delta c^{CRA} < 0$). The dependence of the concentration of saturated KOH vapor on temperature is described by the formula

$$\Delta c^{s}_{KOH}(T) = (A/p)\exp(-B/T),$$

where $A = 1.4 \cdot 10^{10}$ Pa; $B = 1.9 \cdot 10^{4}$°K; p is the pressure of the buffer gas.

Figure 4 shows the dependence of the ratios $c_c(0)/c\ (0)$ and j_{wc}/j_{wa} (the subscript c denotes the cathode, while a denotes the anode) on wall temperature at $p = 10^5$ Pa and $c_\infty/\Phi(\infty) + \Delta c^{CRA}(0) = 0.008$ at the cathode and 0.0057 at the anode (which corresponds to a 40% difference in concentration). At $T_w > 1140$°K, $c_c(0)/c_a(0) = 1.4$, $j_{wc}/j_{wa} = 0$. At 1115°K $< T_w <$ 1140°K, $c_c(0)/c_a(0)$ decreases to unity, while $j_{wc}/j_{wa} = \infty$, since condensation has begun at the cathode but not at the anode. At $T_w < 1115$°K, condensation has also begun at the anode. The concentrations at the cathode and anode become equal, while j_{wc}/j_{wa} decreases from a to the limiting value 1.4 (equal to the ratio of the concentrations before the beginning of condensation). The characteristic range in which there is a sharp change in j_{wc}/j_{wa} (i.e., the saturation of the flows) is $\Delta T \sim T^2_{co}/B \approx 70$°K, where T_{co} is the temperature of the beginning of condensation.

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VAPOR CONDENSATION ON AN INCLINED PLATE WITHIN A POROUS MEDIUM

V. A. Mukhin, V. E. Nakoryakov,
P. T. Petrik, and G. S. Serdakov

UDC 532.546;536.242;536.423.4

In chemical technology, thermal power generation, and other branches of technology the process of heating a surface by condensing vapor is widespread. The process of vapor condensation on smooth surfaces has been studied quite thoroughly [1-6]. Theoretical and experimental studies in this field have expanded the concepts of processes occurring in vapor condensation and permitted development of a technique for engineering calculations of condensation equipment.

Recently a number of technological applications have turned to condensation processes occurring under more complex conditions, for example, in narrow slits, or on surfaces located within porous media. This problem has not been investigated thoroughly. Although the first theoretical treatments have appeared [7, 8], experimental studies of vapor condensation on surfaces located in a porous medium are absent from the literature.

We will consider the problem of vapor condensation on an inclined plane surface located within a porous medium (Fig. 1). The vapor condenses on the outer surface of a moving condensate film.

We make the following basic assumptions: 1) Inertial forces developing in the film are small in comparison to viscous and gravitational forces; 2) there is no friction at the liquid-vapor phase boundary, and the temperature of the outer surface of the condensate film remains constant at the saturation temperature; 3) heat transfer is accomplished by thermal conductivity (effective) of the liquid across the film, while heat transport in the longitudinal direction may be neglected; 4) the physical properties of the liquid are temperature-independent.

To calculate velocity profiles in the liquid film moving along the inclined surface we will use Brinkman's filtration equation [9], which in essence is a simple superposition of Darcy's law and the equation of viscous flow in a porous medium. For planar flow in a porous medium under the action of gravity it has the form

$$\mu' d^2u/dy^2 - \mu' u/\Pi + (\rho' - \rho'')g_\varphi = 0, \qquad (1)$$

with boundary conditions $y = 0$, $u = 0$; $y = \delta$, $du/dy = 0$, where u is the dimensionless velocity of the motion and y is the transverse coordinate; $g_\varphi = g\cos\varphi$; φ is the angle between the surface studied and the vertical; μ', ρ' are the liquid viscosity and density; ρ'' is the vapor density; Π is the permeability of the porous medium and δ is the film thickness. The solution of Eq. (1) is:

Novosibirsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 5, pp. 85-90, September-October, 1985. Original article submitted June 22, 1984.